Invariance property of minimal prime ideals

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Abstract: Let R be a ring. An additive mapping $d : R \to R$ is called derivation of a ring R if d satisfies $d(xy) = d(x)y + xd(y)$, for all $x, y \in R$. A family of additive mappings $(d_i)_{i\in\mathbb{N}}$ from R to itself with d_0 defined as the identity map on R is termed as a higher derivation if $d_n(xy) = \sum_{i+j=n} d_i(x)d_j(y)\ \forall\ x, y \in R$ and $n \geq 1$. In 1988, Herstein suggested a conjecture stating that every minimal prime ideal of a semiprime ring R is invariant under any derivation d of R. Partial results of this conjecture was brought up by many authors (see [Math. Z. (1982), **180**, 503–523] for more details). The best result based on this conjecture was given by Beidar and Mikhalev [Trudy Sem. Petrovski **10**, 227-234]. In 2006, Chuang and Lee [J. Algebra (2006) **302**(1), 305-312] proved the following: if the semiprime ring R either satisfies a polynomial identity or has only countably many elements, then there exists a family $\{P_\alpha\}_{\alpha\in A}$ of minimal prime ideals such that $\bigcap_{\alpha\in A} P_\alpha = 0$ and each P_α is d-invariant for any derivation d of R. In this line, Matczuk [Contemp. Math., Amer. Math. Soc. (2015) **634**, 223-225] proved that every minimal prime ideal P, which has nonzero annihilator of a semiprime ring R is invariant under any derivations d of R which was further generalized by Lee and Lin [Comm. Algebra, **46**(8), 3436-3441] for arbitrary rings.

In this talk, we shall discuss some recent development on the above mentioned conjecture in the setting of rings with involutions involving higher derivations and its related maps.